

# Technical Notes

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## Diffusion Timescale of Porous Pressure-Sensitive Paint

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### Introduction

**P**RESSURE-SENSITIVE paint (PSP) is an optical pressure sensor that utilizes the oxygen quenching of luminescence. PSP measurements in unsteady aerodynamic flows require fast time response of the paint.<sup>1–3</sup> There are two characteristic timescales that are related to the time response of PSP. One is the luminescent lifetime representing an intrinsic physical limit for the achievable temporal resolution of PSP. Another is the timescale of oxygen diffusion across the PSP layer. When the timescale of oxygen diffusion is much larger than the luminescent lifetime, the time response of PSP is controlled by oxygen diffusion. In a thin homogeneous polymer layer where diffusion is Fickian, the oxygen concentration  $[O_2]$  can be described by the diffusion equation in one dimension:

$$\frac{\partial [O_2]}{\partial t} = D_m \frac{\partial^2 [O_2]}{\partial z^2} \quad (1)$$

where  $D_m$  is the diffusivity of oxygen mass transfer in the polymer,  $t$  is the time, and  $z$  is the coordinate normal to the layer. For an arbitrary time-dependent change of the oxygen concentration at the air–paint interface, the complete convolution-type solution of Eq. (1) with the suitable boundary conditions can be obtained by using the Laplace transform (see Refs. 4 and 5). The special trigonometrical-series-type solutions for a step change and a sinusoidal change of oxygen were used for PSP dynamic analysis.<sup>1–3</sup> The solution of the diffusion equation gives a well-known square-law estimate for the diffusion timescale  $\tau_{diff}$  through a PSP layer:

$$\tau_{diff} \propto h^2 / D_m \quad (2)$$

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where  $h$  is the PSP layer thickness. The response time of a homogeneous PSP is proportional to the square of the layer thickness and inversely proportional to the diffusivity of mass transfer  $D_m$ . This estimate is correct for a conventional homogeneous PSP.

Compared with a conventional PSP, a porous PSP has a much shorter diffusion response time ranging from 18 to 500  $\mu$ s (Ref. 6). The shorter diffusion timescale is essentially related to an enlarged air–polymer interface in a porous PSP. Interestingly, recent measurements of the response time for three polymers, GP197, GP197/BaSO<sub>4</sub> mixture, and poly(TMSP), showed that the classical square-law estimate (2) does not hold for porous PSPs.<sup>7,8</sup> As shown in Fig. 1, measurements gave the power-law relations  $\tau_{diff} \propto h^{1.83}$  for GP197,  $\tau_{diff} \propto h^{1.07}$  for GP197/BaSO<sub>4</sub> mixture, and  $\tau_{diff} \propto h^{0.29}$  for poly(TMSP) at a temperature of 313.1 K. For the GP197 silicone polymer, the power-law exponent is close to 2 as predicted by the classical estimate. However, the power-law exponents for the porous polymer materials GP197/BaSO<sub>4</sub> mixture and poly(TMSP) are significantly smaller than 2. In addition, Fig. 2 shows that the

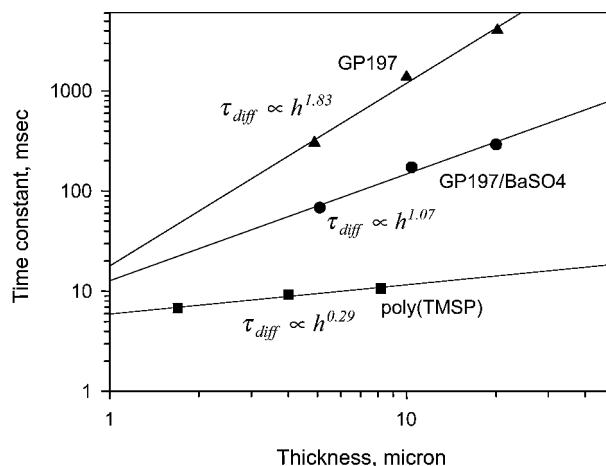


Fig. 1 Power-law relationship between response time and coating thickness for the three polymers GP197, GP197/BaSO<sub>4</sub> mixture, and poly(TMSP) at a temperature of 313.1 K.

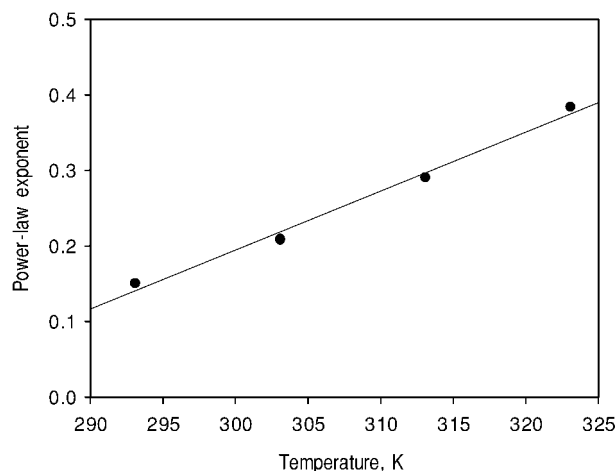


Fig. 2 Exponent of the power-law relation between timescale and coating thickness for the polymer poly(TMSP) as a function of temperature.

power-law exponent for the polymer poly(TMSP) linearly increases with temperature over a temperature range from 293.1 to 323.1 K. Unfortunately, a comprehensive and universal theory of diffusion in porous materials has not been developed yet. To understand the time response of a porous PSP, this Note attempts to derive the expressions for the effective diffusivity and the diffusion response time of a porous polymer layer from a standpoint of phenomenology.

### Effective Diffusivity

Diffusion in a porous material can be considered as a diffusion problem in a two-phase system made up of one disperse phase and one continuous polymer. In PSP, the disperse phase is composed of numerous pores filled with air. An element of a porous polymer layer of length  $l$ , width  $l$ , and thickness  $h$  is considered, as shown in Fig. 3. The coordinate  $z$  is normally directed to the polymer layer from the upper surface of the layer. We assume that many cylindrical (tubelike) pores are distributed in the element, and the pores are oriented in the  $z$  direction. The effective radius and depth of a pore are denoted by  $r_{\text{pore}}$  and  $h_{\text{pore}}$ , respectively. The radius of a pore is much larger than the size of a molecule of oxygen. In general, the depth of a pore is smaller than the layer thickness, that is,  $h_{\text{pore}} \leq h$ . For simplicity of expression, the normal directional derivative of the oxygen concentration  $[O_2]$  at the air-polymer interface is

$$v_n(z) = \frac{\partial [O_2]}{\partial n} \quad (3)$$

The effective diffusivity  $D_{m \text{ eff}}$  of the porous polymer layer with cylindrical pores is given by a balance equation between the mass transfer through the apparent homogenous upper surface and the total mass transfer across the air-polymer interface:

$$D_{m \text{ eff}} l^2 v_n(0) = D_m (l^2 - N_{\text{pore}} \pi r_{\text{pore}}^2) v_n(0) + D_m N_{\text{pore}} \pi r_{\text{pore}}^2 v_n(h_{\text{pore}}) + D_m N_{\text{pore}} 2\pi r_{\text{pore}} \int_0^{h_{\text{pore}}} v_n(z) dz \quad (4)$$

where  $N_{\text{pore}}$  is the total number of the pores in the element and  $D_m$  is the diffusivity of the polymer continuum. The integral term in Eq. (4) is the total mass transfer across the peripheral surface of the pores in the element. Thus, the effective diffusivity  $D_{m \text{ eff}}$  is given by

$$\frac{D_{m \text{ eff}}}{D_m} = 1 + \left[ \frac{v_n(h_{\text{pore}})}{v_n(0)} - 1 \right] N_{\text{pore}} \pi r_{\text{pore}}^2 l^{-2} + N_{\text{pore}} 2\pi r_{\text{pore}} l^{-2} v_n^{-1}(0) \int_0^{h_{\text{pore}}} v_n(z) dz \quad (5)$$

In a simplified case where  $v_n(z) = \text{const}$  across the thin layer, Eq. (5) becomes

$$D_{m \text{ eff}}/D_m = 1 + 2a_v r_{\text{pore}}^{-1} h \quad (6)$$

where  $a_v = N_{\text{pore}} \pi r_{\text{pore}}^2 h_{\text{pore}} l^{-2} h^{-1}$  is the volume fraction of the cylindrical pores in the polymer layer. Equation (6) indicates an

increase of the effective diffusivity that is proportional to the volume fraction of the pores and a ratio between the polymer layer thickness and the radius of the pore. Expression (6) for  $D_{m \text{ eff}}$  is valid only for an ideal porous polymer layer with the straight cylindrical pores oriented normally. Nevertheless, this model should be generalized for real porous polymers, where topology of the pores is often highly complicated.

In a more realistic model, the topological structure of a pore is considered as a highly convoluted and folded tube in the polymer layer while the cross section of the tube remains unchanged. The integral in Eq. (5) should be replaced by the integral along the path of the highly convoluted tubelike pore. In this case, the concept of the fractal dimension should be introduced because the length of the highly convoluted tube is no longer proportional to the linear length scale of the tube in the  $z$  direction, for example,  $h_{\text{pore}}$  (Ref. 9). According to the length-area relation for a fractal path, the integral along the path is proportional to  $A_{\text{pore}}^{d_{fr}/2}$  or  $h_{\text{pore}}^{d_{fr}}$ , where  $d_{fr}$  ( $1 \leq d_{fr} < 2$ ) is the fractal dimension of the path of the pore and  $A_{\text{pore}} \propto h_{\text{pore}}^2$  is the characteristic area covering over the path. Loosely speaking, the fractal dimension represents the degree of complexity of the pore pathway. To take the fractal nature of the pores into account, Eq. (5) is generalized by using a Riemann–Liouville fractional integral of the order  $d_{fr}$ , that is,<sup>10</sup>

$$\frac{D_{m \text{ eff}}}{D_m} = 1 + \left[ \frac{v_n(h_{\text{pore}})}{v_n(0)} - 1 \right] N_{\text{pore}} \pi r_{\text{pore}}^2 l^{-2} + N_{\text{pore}} 2\pi r_{\text{pore}} l^{-2} v_n^{-1}(0) \int_0^{h_{\text{pore}}} v_n(z) (dz)^{d_{fr}} \quad (7)$$

Note that a unitary constant with the dimension  $[m^{1-d_{fr}}]$  is implicitly embedded in the third term in the right-hand side of Eq. (7) to make Eq. (7) dimensionally consistent. This dimensional constant is implicitly contained in all of the results derived from Eq. (7). In a simplified case where  $v_n(z) = \text{const}$  across the thin layer, a generalized expression for  $D_{m \text{ eff}}$  is

$$\frac{D_{m \text{ eff}}}{D_m} = 1 + \frac{2a_v r_{\text{pore}}^{-1}}{\Gamma(1+d_{fr})} \left( \frac{h_{\text{pore}}}{h} \right)^{d_{fr}-1} h^{d_{fr}} \quad (8)$$

where  $\Gamma(1+d_{fr})$  is the gamma function. Here,  $h_{\text{pore}}$  is interpreted as a linear length scale of the convoluted tube in the  $z$  direction and  $a_v$  is the volume fraction of the apparent cylindrical pores. Expression (8) clearly shows that the effective diffusivity  $D_{m \text{ eff}}$  is not only proportional to  $h^{d_{fr}}$ , but also related to the parameters of porosity  $a_v r_{\text{pore}}^{-1}$  and  $h_{\text{pore}}/h$ . For  $d_{fr} = 1$ , Eq. (8) is simply reduced to Eq. (6).

### Diffusion Timescale

For a porous polymer layer where diffusion is Fickian under some microscopic assumptions,<sup>11,12</sup> diffusion equation (1) is still a valid phenomenological model as long as the diffusivity  $D_m$  is replaced by the effective diffusivity  $D_{m \text{ eff}}$ . Hence, the estimate for the diffusion timescale for a porous PSP layer is

$$\tau_{\text{diff}} \propto \frac{h^2/D_m}{1 + [2a_v r_{\text{pore}}^{-1}/\Gamma(1+d_{fr})] (h_{\text{pore}}/h)^{d_{fr}-1} h^{d_{fr}}} \quad (9)$$

Equation (9), a generalized form of Eq. (2), clearly illustrates how the parameters of porosity  $a_v r_{\text{pore}}^{-1}$  and  $h_{\text{pore}}/h$  and the fractal dimension  $d_{fr}$  affect the response time of a porous PSP. For  $a_v r_{\text{pore}}^{-1} \ll 1$  or  $h_{\text{pore}}/h \ll 1$ , Eq. (9) naturally approaches to the classical square-law estimate (2) for a homogenous polymer layer. On the other hand, for  $a_v r_{\text{pore}}^{-1} \gg 1$  and  $h_{\text{pore}}/h \approx 1$ , another asymptotic estimate for  $\tau_{\text{diff}}$  is a simple power law:

$$\tau_{\text{diff}} \propto h^{2-d_{fr}}/D_m \quad (10)$$

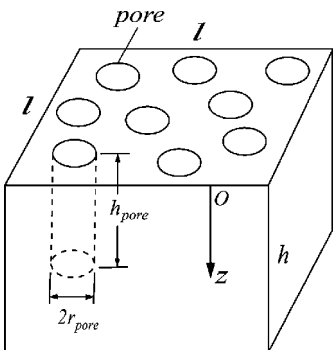


Fig. 3 Element of a porous polymer layer.

Estimate (10) is asymptotically valid for a very porous polymer layer. The exponent in the power-law relation between the response time  $\tau_{\text{diff}}$  and thickness  $h$  deviates from 2 by the fractal dimension  $d_{fr}$  because of the presence of the fractal pores in the polymer layer. Relation (10) provides an explanation for the experimental finding that the exponent  $q$  in the power-law relation  $\tau_{\text{diff}} \propto h^q$  is less than 2 for porous PSPs.<sup>7,8</sup> Also, this relation can serve as a useful tool to extract the fractal dimension of the tubelike pores in a very porous polymer layer from measurements of the diffusion response time. The fractal dimension  $d_{fr}$  of the pore in the polymer poly(TMSP) is  $d_{fr} = 1.71$ . For the GP197/BaSO<sub>4</sub> mixture, the fractal dimension  $d_{fr}$  is close to 1. In addition, based on the experimental results shown in Fig. 2, one knows that the fractal dimension  $d_{fr}$  for the polymer poly(TMSP) linearly decreases with temperature in a temperature range from 293.1 to 323.1 K. This implies that the geometric structure of the pore in poly(TMSP) may be altered by temperature change. Note that the diffusivity  $D_m$  of oxygen mass transfer is also temperature dependent, but it is independent of the coating thickness  $h$ . Therefore, the experimental results in Fig. 2 mainly reflect the effect of temperature on the geometric structure of the pores in the polymer rather than the diffusivity.

### Conclusions

The Note develops a simple phenomenological model for the effective diffusivity of a porous PSP and gives a new expression that clearly illustrates the relationship between the diffusion timescale and the fractal dimension of the pores. The theoretical results can not only explain why a porous PSP is able to achieve a very fast time response, but also quantitatively show how the fractal dimension and the parameters of porosity affect the response time. For very porous PSPs, the classical square-law estimate of the diffusion timescale should be replaced by the generalized relation derived in the Note.

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## Evaluation of Turbulence-Model Performance in Jet Flows

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### Introduction

THE importance of reducing jet noise in both commercial and military aircraft applications has made jet acoustics a significant area of research.<sup>1</sup> A technique for jet-noise prediction commonly employed in practice is the approach of Mani, Giebe, and Balsa (MGB)<sup>2</sup> based on the Lighthill acoustic analogy.<sup>3</sup> This technique requires as aerodynamic input mean flow quantities and turbulence quantities like the kinetic energy and the dissipation. The purpose of the present Note is to assess existing capabilities for predicting these aerodynamic inputs. Two modern Navier–Stokes flow solvers, coupled with several modern turbulence models, are evaluated by comparison with experiment for their ability to predict mean flow properties in a supersonic jet plume. Potential weaknesses are identified for further investigation. Another comparison with similar intent is discussed by Barber et al.<sup>4</sup> The ultimate goal of this research is to develop a reliable flow solver applicable to the low-noise, propulsion-efficient, nozzle exhaust systems being developed in NASA focused programs. These programs address a broad range of complex nozzle geometries operating in high-temperature, compressible, flows.

Seiner et al.<sup>5</sup> previously discussed the jet configuration examined here. This convergent-divergent nozzle with an exit diameter of 3.6 in. was designed for an exhaust Mach number of 2.0 and a total temperature of 915 K. The acoustic and aerodynamic data reported by Seiner et al.<sup>5</sup> covered a range of jet total temperatures from 40 to 1204 K at the fully expanded nozzle pressure ratio. The aerodynamic data included centerline mean velocity and total temperature profiles.

Computations were performed independently with two computational fluid dynamics (CFD) codes, ISAAC<sup>6</sup> and PAB3D.<sup>7</sup> Turbulence models employed include the  $k$ – $\epsilon$  model,<sup>8</sup> the Gatski–Speziale algebraic-stress model,<sup>9</sup> and the Girmaji model,<sup>10</sup> with and without the Sarkar compressibility correction.<sup>11</sup> Centerline values of mean velocity and mean temperature are compared with experimental data.

### Codes and Models

ISAAC (Integrated Solution Algorithm for Arbitrary Configurations)<sup>6</sup> is a finite volume code of second-order accuracy, which solves the full Favre-averaged Navier–Stokes equations. An upwind scheme based on Roe's flux splitting is used for the convective terms, central differencing for the diffusion terms, and an implicit, spatially split, approximate-factorization scheme for iteration.

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